Degrees of Freedom and the Deconfining Phase Transition

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There is a sharp increase in the relative number of degrees of freedom at the deconfining phase transition. Characterizing this increase using the Polyakov Loop model, we find that for a nearly second order deconfining phase transition, the medium-induced energy loss turns on rapidly above T_c, proportional to the relative number of degrees of freedom. Further, energy loss is logarithmically dependent on the screening mass, and thus is sensitive to nearly critical scattering.

Experiments indicate that for the central collisions of large nuclei, $A \sim 200$, there are marked changes between energies of $\sqrt{s}/A = 17$ GeV, at the SPS, and 130 GeV, at RHIC [1]. Comparing central AA collisions to pp, the spectrum of semi-hard particles is rather different. At the SPS, in AA the hard p_t spectrum, scaled by the number of binary collisions, is enhanced over pp. At RHIC, the opposite is true: the semi-hard p_t spectrum per nucleon-nucleon collision, is suppressed in central AA, relative either to peripheral AA, or $p\bar{p}$ [2]. This could be the result of "energy loss" [3–5], where a fast colored field loses energy as it passes through a thermal bath. In peripheral AA collisions, secondary hadrons are distributed anisotropically in the transverse momentum p_t [6]. Experimentally, this azimuthal anisotropy increases with p_t until $p_t \sim 2$ GeV, at which point it flattens [7]. This flattening may also be due to energy loss [8].

In the limit of infinitely large nuclei, $A \to \infty$, it is plausible that the initial energy density produced in a central AA collision — at a fixed value of $\sqrt{s}/A$ — evolves into a system in equilibrium at a temperature T. With great optimism, assuming that $A \sim 200$ is near $A = \infty$, one might imagine that the difference between SPS and RHIC is because temperatures reached at RHIC exceed T_c, the critical temperature for QCD.

Thus it is of interest to know how quantities change as one goes through the phase transition. In this paper we give an analysis in terms of the Polyakov Loop model [9–11].

In QCD, there is a large increase in the number of degrees of freedom at the deconfining phase transition. We count degrees of freedom as appropriate for the pressure of free, massless fields at nonzero temperature, so if each boson counts as one, then each fermion counts as 7/8. In the hadronic phase, pions contribute $c_\pi = 3$ ideal degrees of freedom. By asymptotic freedom, at infinite temperature QCD with three flavors of quarks is an Ideal Quark-Gluon Plasma, with $c_{\rm QGP} = 47\,\frac{1}{2}$ degrees of freedom. This is an increase of more than a factor of ten.

To measure the change in the number of degrees of freedom, we introduce the relative pressure, $n(T)$: at a temperature T, this is the ratio of the true pressure, $p(T)$, to that of an Ideal Quark-Gluon Plasma, $p_{\rm ideal} = c_{\rm QGP}(\pi^2/90)T^4$:

$$n(T) \;\equiv \frac{p(T)}{p_{\rm ideal}} \quad . \tag{1}$$

By asymptotic freedom, QCD is an ideal gas at infinite temperature, and so

$$n(\infty) = 1 \; . \tag{2}$$

For $T < \infty$, corrections to ideality are determined by the QCD coupling constant, $\alpha_s \propto 1/\log(T)$, with $n(T) - 1 \propto -\alpha_s$ [12].

For an *exact* chiral symmetry which is spontaneously broken by the vacuum, about zero temperature the free energy is that of free, massless pions. Thus at zero temperature, the relative pressure is the ratio of the ideal gas coefficients [13]:

$$n(0) \; = \; \frac{c_\pi}{c_{\rm QGP}} \; . \tag{3}$$

At low temperature, corrections to ideality are given by chiral perturbation theory for massless pions, $n(T) - n(0) \sim +(T/f_\pi)^4 \; n(0)$, with f_π the pion decay constant. In QCD, pions are massive, and the relative pressure is Boltzmann suppressed at low temperature, $n(T) \sim \exp(-m_\pi/T)(m_\pi/T)^{5/2}$, so $n(0) = 0$.

Given the great disparity between c_π and $c_{\rm QGP}$, consider an approximation where the hadronic degrees of freedom are neglected relative to those of the deconfined phase [14]. Then the relative pressure vanishes throughout the hadronic phase, $n(T) = 0$ for $T < T_c$. The question is then: how does the relative pressure go from zero at T_c, when deconfinement occurs, to near one at higher T?

This can be answered by numerical simulations of Lattice QCD [15]. Consider first quenched QCD, with pure glue and no dynamical quarks, which is close to the continuum limit [16]. For three colors, the Lattice finds no measurable pressure in the hadronic phase (glueballs are heavy), so our approximation of $n(T) = 0$ when $T < T_c$ is good. $n(T)$ increases quickly above T_c, and is $\sim .8$ by $T \sim 2T_c$. To characterize the change in the relative

pressure, consider the ratio of $e - 3p$, where $e(T)$ is the true energy density of QCD, to the energy of an Ideal Quark-Gluon Plasma, $e_{\rm ideal} = 3\, p_{\rm ideal}$:

$$\frac{e - 3p}{e_{\rm ideal}} = \frac{T}{3} \frac{\partial n}{\partial T} \ . \tag{4}$$

Lattice simulations find that this ratio has a sharp "bump" at $\sim 1.1 T_c$, suggesting that the relative pressure changes quickly, when the reduced temperature,

$$t \equiv \frac{T}{T_c} - 1 \ , \tag{5}$$

is small, $t \sim .1$.

The Lattice is more uncertain with dynamical quarks. The pions are too heavy, and it is not near the continuum limit. So far, the Lattice finds that $n(T/T_c)$ is about the same with dynamical quarks as without [15,17]. This suggests that the pure glue theory may be a reasonable guide to how the relative pressure increases above T_c. The approximate universality of $n(T/T_c)$ is remarkable. At present, the Lattice finds no true phase transition in QCD, with T_c smaller by $\sim .6$ than in the quenched theory [15]. Indeed, even the ideal gas coefficients are very different: $c_{\rm QGP}$ is only 16 in the quenched theory, versus $47\,\frac{1}{2}$ in QCD.

The greatest change with dynamical quarks is a small, but measurable, pressure in the hadronic phase. While in the quenched theory $n(T) \sim 0$ for $T < T_c$, with dynamical quarks, although $n(0) \sim 0$, there is a nonzero relative pressure at the critical temperature, with $n(T_c) \sim .1$ [15]. Indeed, with no true phase transition, an approximate T_c can only be defined as the point where the relative pressure increases sharply, reaching $n \sim .8$ by $2T_c$ [15].

The Polyakov Loop model [9–11] is a mean field theory for the relative pressure. In a pure glue theory, the expectation value of the Polyakov Loop, $\ell_0(T)$, behaves like the relative pressure: it vanishes when $T < T_c$, and is nonzero above T_c. Indeed, again by asymptotic freedom, $\ell_0 \to 1$ as $T \to \infty$. The simplest guess for a potential for the Polyakov Loop is:

$$V(\ell) = -\frac{b_2}{2} |\ell|^2 + \frac{1}{4} \left(|\ell|^2 \right)^2 \ . \tag{6}$$

Defining ℓ_0 as the minimum of $V(\ell)$ for a given $b_2(T)$, the relative pressure is given by [9–11]:

$$n(T) = -4 V(\ell_0) = \ell_0^4 \ ; \tag{7}$$

$b_2 > 0$ above T_c ($b_2(T) \to 1$ for $t \to \infty$), and < 0 below T_c. Thus if the relative pressure changes when the reduced temperature $t \sim .1$, the change for $\ell_0(T) \sim n^{1/4}$ is even more rapid, within 2.5% of T_c.

For two colors, (6) is a mean field theory for a second order deconfining transition [18]. The ℓ field is real, and so the potential defines a mass: $(m_\ell/T)^2 = (1/Z_s) \partial^2 V/\partial \ell^2$, with

$$m_\ell(T)/T \propto \ell_0 \sim n^{1/4} \ , \tag{8}$$

where Z_s is the wave function normalization constant for ℓ, $Z_s = 3/g^2$, up to corrections of order g^0 [19]. This is measured from the two point function of Polyakov loops in coordinate space, $\propto (1/r) \exp(-m_\ell r)$ as $r \to \infty$.

For three colors, ℓ is a complex valued field, and a term cubic in ℓ appears in $V(\ell)$, $-b_3(\ell^3 + \ell^{*3})/6$. This produces a first order deconfining transition, where ℓ_0 jumps from 0 at T_c^- to $\ell_c = 2b_3/3$ at T_c^+ [10]. The ℓ field has two masses, from its real (m_ℓ) and imaginary ($\widetilde{m}_\ell$) parts. At T_c^+, $\sqrt{Z_s} m_\ell/T = \ell_c$; from the Lattice, $\sqrt{Z_s} m_\ell/T \sim .3$ [15], which gives $b_3 \sim .45$. This small value of b_3 reflects the weakly first order deconfining transition for three colors [15,16]. The mass for the imaginary part of ℓ is $\sqrt{Z_s} \widetilde{m}_\ell(T)/T \propto \sqrt{b_3 \ell} \sim n^{1/8}$; at T_c^+, $\widetilde{m}_\ell/m_\ell = 3$. With dynamical quarks, in principle a term linear in ℓ, $-b_1(\ell + \ell^*)/2$, can also appear in $V(\ell)$ [20]. If the pion pressure is included below T_c, however, b_1 is very small, $\leq .03$.

Thinking of ℓ_0 provides a useful way of viewing the deconfining phase transition. For a strongly first order transition — as appears to occur for four or more colors [21] — ℓ_0, jumps from zero below T_c, to a value near one just above T_c. As ℓ_0 is near one, the deconfined phase is presumably well described as a nearly Ideal Quark-Gluon Plasma [22]. In this case, there is a hadronic phase below T_c, and a Quark-Gluon Plasma from T_c immediately on up.

In contrast, for three colors the deconfining transition is weakly first order. As the energy density is discontinuous at T_c, for small t the relative pressure is linear in the reduced temperature,

$$n(T) \ \sim \ 3\, r\, t \ ; \tag{9}$$

here $r \equiv e(T_c^+)/e_{\rm ideal}(T_c)$ is the ratio of the energies at T_c, in the deconfined phase versus an Ideal Quark-Gluon Plasma. For quenched QCD, $r \sim 1/3$ [16], which gives $n(T) \sim t$, and so $\ell_0(T) \propto t^{1/4}$. Except very near T_c, this simple estimate agrees with more complicated analysis using $b_3 \neq 0$ [10,11]. For example, at only 5% above T_c, this estimate gives $\ell_0 \sim .05^{1/4} \sim .5$. For three colors, then, there is a (non)-Ideal Quark-Gluon Plasma only at temperatures above $\sim 2T_c$; between T_c and $\sim 2T_c$, the Polyakov Loop dominates the free energy, going from $\sim .5$ at $1.05 T_c$ to ~ 1 by $2T_c$.

The difference between these two scenarios: a strongly first order transition, where $\ell_0(T)$ is approximately constant above T_c, and nearly second order behavior, where $\ell_0(T)$ changes significantly, is in principle observable. As an example, consider energy loss for a fast parton, with a high energy E. We first give a general discussion of energy loss in a medium [4,5], and then discuss the differences between a strong first order transition, and one which is nearly second order.

We introduce the energy scale [5],

$$E_{\rm cr} = \frac{m_\ell^2}{\lambda}\, L^2 \ , \tag{10}$$

where λ is the mean free path and L is the thickness of the medium. The high-energy jet loses energy by radiating gluons with energy $\omega < E$. There are several contributions to the total energy loss of the jet, ΔE, depending on the energy of the radiation. For the contribution from $\omega > E_{\rm cr}$, which exists if $E > E_{\rm cr}$, effectively only one single scattering occurs (this is the so-called factorization regime) and so that contribution is medium independent [5]. In what follows we rather focus on the medium-induced energy loss, from the region where ω is less than $E_{\rm cr}$.

For very small frequency, $\omega < E_{\rm LPM} \equiv \lambda m_\ell^2$, the formation time [3–5] $t_f \sim \omega/m_\ell^2$ of the radiation from the hard jet is short, and so incoherent radiation takes place. This is the so-called Bethe-Heitler regime; the contribution to ΔE is just a sum from single scatterings on L/λ scattering centers. In the high-energy limit $E, E_{\rm cr} \gg E_{\rm LPM}$ the region of phase space with $\omega < E_{\rm LPM}$ contributes little to ΔE and will be neglected.

The largest contribution is rather from the Landau-Pomeranchuk-Migdal (LPM) regime, where successive scatterings coherently interfere [3–5]. Integrating the radiation intensity distribution over ω from zero to some energy E^* yields a total energy loss of [4,5]

$$- \Delta E \sim \frac{3\,\alpha_s}{\pi} \, \sqrt{E_{\rm cr} E^*} \, \log \frac{2E}{L m_\ell^2} \; . \qquad (11)$$

There is a logarithmic sensitivity to the infrared scale m_ℓ [4,23]. When the jet energy E is less than the factorization scale $E_{\rm cr}$, we can integrate ω all the way up to $E^* = E$, so

$$- \Delta E \; \sim \; \frac{3\,\alpha_s}{\pi} \, \sqrt{E\, E_{\rm cr}} \, \log \frac{2E}{L m_\ell^2} \quad , \quad (E < E_{\rm cr}) \; . \qquad (12)$$

Note that $-\Delta E$ should not exceed E. This requires that m_ℓ is not so small that the logarithm overwhelms the $\sim \alpha_s$. However, $E_{\rm cr}$ is small near T_c, so minijets with energies of at least a few GeV are above $E_{\rm cr}$ anyways, and eq. (12) does not apply.

Rather, for jet energies greater than $E_{\rm cr}$, the total medium-induced energy loss is given by integrating over ω up to the factorization scale $E_{\rm cr}$; setting $E^* = E_{\rm cr}$ in (11),

$$- \Delta E \; \sim \; \frac{3\,\alpha_s}{\pi} \, E_{\rm cr} \, \log \frac{2E}{L m_\ell^2} \quad , \quad (E > E_{\rm cr}) \; . \qquad (13)$$

To compute the critical energy $E_{\rm cr}$, we need the inverse mean free path, λ^{-1}. This is approximately the product of the density, ρ, times the elastic cross section, $\sigma_{\rm el}$. The elastic cross section is quadratically divergent in the infrared. This divergence is naturally cut off by m_ℓ, so the elastic cross section $\sigma_{\rm el} \propto \alpha_s^2/m_\ell^2$, and

$$\frac{m_\ell^2}{\lambda} \propto \rho \; . \qquad (14)$$

The scale $E_{\rm cr}$ is then proportional to ρ; this follows automatically from our assumption that $\lambda^{-1} \sim \rho \sigma_{\rm el}$. Thus in the high-energy regime above $E_{\rm cr}$, ignoring the logarithmic dependence upon m_ℓ, energy loss is proportional to ρ; below that scale, to $\sqrt{\rho}$. This has been emphasized by Baier, Dokshitzer, Mueller, and Schiff [5].

Before giving estimates of m_ℓ and λ, we can understand how energy loss changes, depending upon the order of the deconfining phase transition. For a nearly second order transition, m_ℓ/T is small near T_c, eq. (8), and then increases rapidly. As the energy loss ΔE depends logarithmically on m_ℓ, for small m_ℓ energy loss is enhanced. This is directly analogous to critical opalescence. In contrast, for a strongly first order transition, m_ℓ is large at T_c^+, with m_ℓ/T approximately constant with increasing temperature.

What are reasonable values for m_ℓ and λ ? In the extreme perturbative regime, $T \gg T_c$, (static) electric fields are heavy, with a mass $\propto \sqrt{\alpha_s} T$, while the static magnetic fields are light, $m_{mag} \propto \alpha_s T$. The inverse mean free path, λ^{-1}, equals the damping rate for a gluon with momentum $\sim T$, and is $\propto \alpha_s T$.

At temperatures $\sim 2T_c$, this ordering is reversed, as static electric fields are significantly lighter than static magnetic fields: $m_\ell \sim 2.5T$, while the static mass for magnetic glueballs is $m_{mag} \sim 6T$ [24]. There are no estimates of the damping rate; we guess that $\gamma \sim \lambda^{-1} \sim T$. This seems reasonable for a quasiparticle with such a mass, as $\gamma/m_\ell \sim 1/2.5 = .4$ is less than one. If the width were much larger, then it would not make sense to speak of quasiparticles. Conversely, in a strongly coupled system, it is unreasonable to think that the width could be much smaller than the mass.

In the derivation of energy loss, implicitly it is assumed that multiple scatterings of the hard jet are independent of each other. This requires that the range of the potential is smaller than the mean free path, $m_\ell^{-1} < \lambda$ [4,5], which is equivalent to having quasiparticles with relatively narrow width.

At $2T_c$, then, $m_\ell^2/\lambda \sim (2.5T)^2 T$. Below $2T_c$, by our mean field analysis, then, $m_\ell^2/\lambda \sim 6.25 T^3 n(T)$. Notice that as $m_\ell^2 \sim \sqrt{n}\, T^2$, and $\gamma = \lambda^{-1} \sim \sqrt{n}\, T$, that $\gamma/m_\ell \sim n^{1/4}$: the ℓ quasiparticles become narrower as $T \to T_c^+$. Consequently, for temperatures near T_c,

$$E_{\rm cr} \sim 20.{\rm GeV} \times n(T) \left(\frac{T}{T_c} \right)^3 \qquad (15)$$

This number is a *very* crude estimate, obtained by taking $L \sim 5$ fm and $T_c \sim 175$ MeV. For QCD, $n(T_c) \sim .1$, so then $E_{\rm cr} \sim 2$ GeV.

It is interesting to note that while the system is becoming increasingly dilute near T_c for a nearly second order transition, typical minijets with E on the order of several GeV automatically have energies larger than $E_{\rm cr}$. This implies that i) some part of their energy loss is medium independent, from the factorization regime $\omega > E_{\rm cr}$; and that ii) their medium-induced energy loss is

given by eq. (13), not (12). Physically, this is because the formation time $t_f \sim \omega/m_\ell^2$ of the radiation from the hard jet grows like $n^{-1/2}$ as $T \to T_c^+$. In all, $E_{\rm cr}$ is small near T_c, and the regime where ΔE scales with $\sqrt{\rho}$ shrinks. That regime only emerges as $E_{\rm cr}$ increases: by $2T_c$, with $n(2T_c) \sim 1$, $E_{\rm cr}$ has risen dramatically, to ~ 160 GeV ! Thus, the energy loss (13) turns on very rapidly as T increases from T_c. In part this is because of the factor of T^3 in eq. (15), which is present whatever the order of the phase transition. For gauge theories with three colors, such as QCD, however, *additionally* there is an increase $\propto n(T)$ [15–17].

With these numbers, the logarithmic sensitivity to the changes in the screening mass can be significant. For $E = 25$ GeV, $L = 5$ fm, $T = T_c$, then if $m_\ell \sim 2.5T_c$, $\log(2E/(Lm_\ell^2)) \sim 2.3$. Including the change in the screening mass near T_c, with $m_\ell \sim 2.5T_c n^{1/4}$, the logarithm changes to $\log(2E/(Lm_\ell^2)) \sim 3.5$. This is an increase by about 50%. At smaller E, the sensitivity to m_ℓ is even stronger.

Even with the limitations of our approximations, it is clear that since the density vanishes as $T \to T_c^+$, *any* contribution from the deconfined phase vanishes like *some* power of $n(T)$ [25]. For example, estimates indicate that dilepton production from the (nearly ideal) deconfined phase [26] is about as large as that from the hadronic phase [27]. Near T_c, dilepton production from the deconfined phase should be strongly suppressed, quadratically in the density:

$$\frac{dN_{e^+e^-}}{d^4x} \propto n^2\, T^4 \; . \tag{16}$$

This assumes that the density scales as the relative pressure, $n(T)$. Whatever the exact form, it is clear that dileptons from the deconfined phase will turn on later (in T, or $\sqrt{s}/A$) than energy loss, which close to T_c is linear in the density, and thus in $n(T)$.

The amplitude for dilepton production involves a virtual photon off the light cone; this has no power like infrared divergences at either leading or next to leading order in α_s, and so dilepton production should not be especially sensitive to the screening mass, m_ℓ. This may not be true for the production of real photons; this involves processes on the light cone, with severe infrared divergences at next to leading order [28]. At leading order in α_s, real photons from the deconfined phase will be suppressed like $\sim n^2$, as in eq. (16). It is conceivable that infrared effects from higher order diagrams change this to $\sim n^2/m_\ell^2 \sim n^{3/2}$. If true, then there is a sequential series of effects: increasing T from T_c, as the relative pressure increases, first energy loss turns on, then perhaps real photons, then dileptons.

In closing, we remark that changes in the screening mass could also be significant for the collision rates of quarks and gluons. This is particularly true for quantities like the elastic cross section, the color conductivity, *etc*, which are sensitive to m_ℓ [29].

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